TOPIC 0: THE TIME VALUE OF MONEY

In this topic you will learn how to value cash flows. Understanding how to value cash flows is a fundamental skill that will be used throughout this unit and the remainder of your Master of Applied Finance degree. We first look at how to value a single cash flow and introduce the important ideas of compounding and discounting. Next, we consider how to value a stream of cash flows and discuss perpetuities and annuities. Finally, we explore how to value cash flows when interest rates have different compounding frequencies.

This topic is pre-course material. It will not be covered during lectures. You will find numerous questions throughout this topic that you can work on as you read through the topic. There are also additional questions in the problem set at the end of the topic. Answers to all of these questions can be found at:

http://www.mafc.mq.edu.au/applications/minimum-knowledge-requirement/pre-course-materials1/.

1 Parts of these lecture notes may be based on material prepared by Bernd Luedcke. These lecture notes have also benefited from discussions with Frank Ashe, Rob Trevor and Peter Vann.
0.1 Money has time value

Would you rather receive a dollar today or receive a dollar in one year from now?

Obviously, you would prefer to receive the dollar today. The assumption implicit in this common sense choice is that having the use of money for a period of time has value. The earlier receipt of a dollar is more valuable than a later receipt because you can deposit your money today into the bank and earn interest, and thereby get back more money in the future.\(^2\) We call the difference in value between money today and money in the future the *time value of money*. This is one of the most basic and important concepts in finance.

The fact that money has time value means that it is meaningless to compare or combine cash flows that occur at different points in time. To compare or combine cash flows that occur at different points in time, you first need to value them at the same point in time. This means that arithmetic statements like \(C_t + C_s\) and \(C_t - C_s\) make no sense.\(^3\) But the following arithmetic statement does make sense:

\[
V_u(C_t) + V_u(C_s)
\]

where: \(V_u(C_t)\) is the *value* at date \(u\) of \(C_t\) and \(V_u(C_s)\) is the *value* at date \(u\) of \(C_s\).

The important point to note here is that both cash flows are *valued* at the *same* point in time, \(u\).

In the following sections we will go into considerable detail analysing how to value cash flows that occur at different points in time.

\(^2\) This of course assumes the interest rate is positive.

\(^3\) In these arithmetic statements, \(C_t\) and \(C_s\) are two cash flows that that occur at dates \(t\) and \(s\), where \(t\) and \(s\) are not the same date. For example, $1m on the 1st of January 2015 and $1.5m on the 1st of January 2020.
0.2 Setting up the cash flows and the time line in a financial calculator

It is important to fully understand the workings of the following five buttons on your financial calculator:

\[
\begin{align*}
\text{N} &= \text{number of payment periods} \\
\text{I\%YR}^4 &= \text{per payment period effective interest rate} \\
\text{PV} &= \text{present value} \\
\text{PMT} &= \text{recurring periodic payment} \\
\text{FV} &= \text{future value}
\end{align*}
\]

These five buttons allow you to enter and value cash flows using a conceptual time line.

We will adopt the following conventions in setting up time lines:

(a) *Cash flows* occur at the end of each *time period* (unless otherwise stated);
(b) *Cash outflows* are entered as *negative* (-ve) values;
(c) *Cash inflows* are entered as *positive* (+ve) values;
(d) *Today* is represented by \( t=0 \);
(e) The *compounding period* is the same as the *payment period*.

*To think about:* What do all those italicised terms *mean*?

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^4 The \( \text{I\%YR} \) button interacts with the number of periods per year, \( \text{P/YR} \) button. I recommend that you set the \( \text{P/YR} \) button to 1. Other buttons to understand are \( \text{BEG} \) and \( \text{END} \). At this stage, you should set your financial calculator to \( \text{END} \) mode.
The picture below should give you the idea.

![Figure 0.1](image)

**0.3 Valuing a single cash flow**

We begin this section by calculating the *future value* of single cash flow. We then proceed to calculate the *present value* of single cash flow. A single cash flow is sometimes called a *bullet*. In bond markets a long-dated single cash flow is called a *zero coupon bond* or a *deep discount bond*. In other, shorter-dated markets (say, with one year or less to maturity) a single cash flow is called a *bank bill* or a *promissory note*, depending on the issuer.
0.3.1 Future value and compounding

*Future value* is the amount of money a cash flow will grow to at some time in the future by earning interest at some interest rate. The process of calculating the future value of a cash flow is known as *compounding*.

For example, suppose you deposit $1,000 into a bank account for two years. If the annual effective interest rate is 10% the future value of that deposit at the end of year one is:

$$1,000 \times (1 + 0.10) = 1,100,$$

and the future value of the deposit at the end of year two:

$$1,000 \times (1 + 0.10) \times (1 + 0.10) = 1,000 \times (1 + 0.10)^2 = 1,210.$$

Notice that the future value of $1,210 can be broken down into three components:

- The original deposit of $1,000.
- The interest on this deposit of $100 in year one and another $100 in year two. The interest on the original deposit is called *simple interest*.
- In year two you also earn interest of $10 on the $100 interest you received in year one. The effect of *earning interest on interest* is known as *compound interest*.

In general, given a per period effective interest rate $r$, the future value, $FV$, in $n$ periods of a cash flow $C_0$ to be paid or received today, is

$$FV_n = C_0 (1 + r)^n.$$

Notice that $FV_n$ depends upon (is a *function of*) $C_0$, $r$ and $n$. If any of these three were to change, so would $FV_n$.

*To think about:* What happens to $FV_n$ as

- $r \rightarrow \infty$?
- $n \rightarrow \infty$?
Questions

1. You deposit $687,436.81 into a bank account for 7 years. The relevant annual effective interest rate is 5.5%. How much money will be in your account at the end of 7 years?

2. What does the time line look like for Question 1?
0.3.2 Present value and discounting

The calculation of a present value is the reverse of the future value calculation. When calculating present values we are asking “what amount would we need to invest today to have a certain amount in the future”. The process of calculating the present value of a future cash flow is known as discounting.

For example, suppose you want to have $1,000 two years from now and the annual effective interest rate is 10%. The amount you must invest must invest today is:

\[
\frac{1,000}{(1+0.10)^2} = 826.45.
\]

Thus, if you invested $826.45 for two years at an effective annual interest rate of 10% it would grow to $1,000 at the end of year two.

In general, given a per period effective interest rate \( r \), the present value, \( PV \), of a cash flow \( C_n \) to be paid or received in \( n \) periods from now is:

\[
PV = \frac{C_n}{(1+r)^n}.
\]

Notice that \( PV \) depends upon (is a function of) \( C_n \), \( r \) and \( n \). If any of these three were to change, so would \( PV \).

**To think about:** What happens to \( PV \) as
\[
r \to \infty?
\]
\[
n \to \infty?
\]
Question

3. Someone promises to pay you $1,000,000 7 years from now. The relevant annual effective interest rate is 5.5%. What is the present value of this promise?

To think about: What is the most you would pay for this promise today? Why?

What is the least amount the seller of this promise would sell the promise for today? Why?

For you and the seller to trade with each other what price would you both agree on? Why?

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5 Notice that this promise is a financial contract.
0.3.3 Some more thoughts

In sections 0.3.1 and 0.3.2 we looked at calculating the future value and present value of cash flows. In general, we can think of a cash flow as a value – an amount of cash today is a present value and an amount of cash in the future is a future value. Hence we can conclude that:

\[ FV_n = PV (1+r)^n, \]

and

\[ PV = \frac{FV_n}{(1+r)^n}. \]

Notice that \( FV \) and \( PV \) are linked by the number of periods, \( n \), and the effective per period interest rate, \( r \). On many occasions in this unit (and indeed in other units) we will be interested computing the effective per period interest rate from known \( PV \)'s and \( FV \)'s. Using either formula above, we can solve for the effective per period interest rate, \( r \):

\[ r = \left( \frac{FV_n}{PV} \right)^{\frac{1}{n}} - 1. \]

Question

4. Using your answers from questions 1 and 3, show that the annual effective interest rate is 5.5%.
0.4 Valuing a stream of cash flows

Many financial instruments have multiple cash flows that occur at different points in time. In section 0.3 we looked at valuing single cash flows. Now we show how to value a stream of cash flows. Consider a stream of cash flows: \( C_0 \) today, \( C_1 \) at period 1, \( C_2 \) at period 2, and so on up to \( C_N \) at period \( N \). The present value of this cash flow stream is:

\[
PV = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_N}{(1+r)^N} = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n}
\]

Notice that the present value of a stream of cash flows is just the sum of the present values of each cash flow.\(^6\) This approach allows you to value any cash flow stream, regardless of whether the cash flows are equal or unequal. In the next few sections we learn shortcuts for valuing a stream of equal cash flows.

0.4.1 Perpetuities

A stream of equal cash flows that go on forever is called a perpetuity. A financial instrument which has this cash flow pattern is called a consol. The British government issued such instruments until December 1926. An investor purchasing a consol is entitled to receive interest from the British government every year forever. Since these instruments never mature they are still held by investors, albeit infrequently traded.

We could value a perpetuity by applying the present value formula

\[
PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots = \sum_{n=1}^{\infty} \frac{C}{(1+r)^n}
\]

However, this approach would take you forever – literally! Fortunately there is a shortcut that can be used to value a perpetuity.

\(^6\) Note that \((1+r)^0 = 1\), so that \(\frac{C_n}{(1+r)^0} = C_n\).
The present value, $PV$, of a perpetuity that pays a cash flow $C$ at the end of every period and with a per period effective interest rate $r$ is:

$$PV = \frac{C}{r}$$

Like any financial instrument, the present value of a perpetuity is the present value of all of its future cash flows. That is, the present value of a perpetuity is an amount of money that, if you had it today, would enable you to create the same set of cash flows as a perpetuity. For example, suppose you invest an amount $P$ in a bank account. At the end of every period you can withdraw the interest you have earned, $C = r \times P$, leaving the principal, $P$ in the bank. The present value of receiving $C$ in perpetuity is therefore the original principal that was invested, $P = \frac{C}{r}$. Of course, we would have to assume that you can do this forever!

Questions

5. What is the present value of $5,000 to be received at the end of every year in perpetuity if the relevant annual effective interest rate is 5%?
6. If somebody gave you the amount which is your answer to Question 5, how would you convert it back into a perpetuity? (assume that the 5% annual effective interest rate is available to you).

0.4.2 Growing perpetuities

Suppose the cash flows on a perpetuity grow at rate \( g \). The first cash flow is \( C_1 \) and rate \( g \) applies to every cash flow after that so that after \( n \) periods the cash flow is \( C_n = C_1 (1+g)^{n-1} \). The present value of this infinite series of growing cash flows is:

\[
P V = \frac{C_1}{r-g} \quad (r > g)
\]

where:
- \( PV \) is the present value
- \( C_1 \) is the first cash flow, occurring one period from today
- \( r \) is the per period effective interest rate
- \( g \) is the growth rate of the cash flows.

Notice that if \( g = 0 \) the above valuation formula collapses to that for the previous "vanilla" perpetuity.
Perpetuities and growing perpetuities may seem a bit strange to you. You will see the power of these valuation techniques when valuing shares in the Investments unit.

### 0.4.3 Annuities

An annuity is a finite stream of equal cash flows paid at regular intervals. Annuities are amongst the most common kinds of financial instruments. For example, many car loans, leases, pensions, and bonds are annuities.

We could value an $N$-period annuity by applying the present value formula

\[
PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^N} = \sum_{n=1}^{N} \frac{C}{(1+r)^n}.
\]

There is also a shortcut that can be used to value an $N$-period annuity.

The present value, $PV$, of an $N$-period annuity that pays a cash flow $C$ at the end of every period and with a per period effective interest rate $r$ is:

\[
PV = C \times \frac{1}{r} \left(1 - \frac{1}{(1+r)^N}\right)
\]

An $N$-period annuity can be viewed as equivalent to a perpetuity with the first payment made at the end of the first period less another perpetuity whose first payment is made at the end of period $N+1$. The immediate perpetuity net of the delayed perpetuity provides exactly $N$-periods.
Questions

7. Someone promises to pay you $1,000,000 every year (at the end of the year) for the next 15 years. The first payment will be one year from now. The relevant annual effective interest rate is 7.0%. What would you pay, today, for this promise?

8. What does the time line look like for Question 7?
9. Show that your answer in Question 7 is the same as the difference between the present value of the following two perpetuities. Perpetuity 1 pays $1,000,000 at the end every year with the first payment made in 1-year. Perpetuity 2 pays $1,000,000 at the end every year with the first payment made in 16-years. The relevant annual effective interest rate is 7.0%.

10. How would your answer in Question 7 change if the payments start today?
The future value, $FV$, of an $N$-period annuity that pays a cash flow $C$ at the end of every period and with a per period effective interest rate $r$ is:

$$FV = C \times \frac{1}{r} \left( (1+r)^N - 1 \right)$$

Questions

11. Using the same data as in Question 7, what would that series of payments accumulate to at the end of year 15?

12. What is the relationship between the answer to Question 7 and the answer to Question 11?
0.4.4 Growing annuities

Suppose the cash flows on an \( N \)-period annuity grow at rate \( g \). The first cash flow is \( C_1 \) and rate \( g \) applies to every cash flow after that so that after \( n \) time periods the cash flow is \( CF_n = C_1 (1+g)^{n-1} \). The present value of this finite series of growing cash flows is:

\[
PV = C_1 \times \frac{1}{r-g} \left( 1 - \frac{(1+g)^N}{1+r} \right) \quad (r > g)
\]

where:
- \( PV \) is the present value
- \( C_1 \) is the first cash flow, occurring one payment date from today
- \( r \) is the per period effective interest rate
- \( g \) is the growth rate of the cash flows
- \( N \) is the number of periods.

Notice that if \( g = 0 \) the above valuation formula collapses to that for the previous “vanilla” annuity.

Growing annuities aren't all that common. They sometimes arise in the context of valuing mineral extraction projects where the price for the extracted ore, or the costs of key inputs, is assumed to grow at some conjectured rate.
0.5 Different compounding periods

Read McDonald pages 853-857

Here’s yet another consequence of the time value of money: It matters, from the point of view of value, when and how often you receive, or have to pay, an amount of cash. Time is money!!

It is important to remember this in the context of interest rates and how they are quoted. Every interest rate quoted has a compounding frequency associated with it. If this is not stated explicitly confusion can (and often does) arise about exactly what that interest rate means.

For example, imagine a bank pays a rate of 10% per annum compounding semi-annually. This means that interest is paid twice per year. A deposit of $1,000 in the bank would be worth $1,000\times(1+0.05) = $1,050 after six months and $1,050\times(1+0.05) = 1,102.50 at the end of year.

The future value of the deposit in one year can be written as

$$1,000\left(1 + \frac{0.10}{2}\right)^2 = 1,000\times(1+0.05)^2 = 1,102.50.$$  

Now, if you deposited $1,000 at 10% per annum compounding annually (i.e., an annual effective rate) the deposit would be worth $1,000\times(1+0.10) = $1,100 at the end of the year. Notice that the future value at the end of the year is greater with semi-annual compounding than annual compounding. With semi-annual compounding, interest is paid twice a year. It’s important to remember that the interest earned in the first six months also earns interest. This is the interest on interest effect, which is what the notion of compounding is all about.

Because $1,000\times(1+0.05)^2 = 1,000\times1.1025 = 1,102.50$, a rate of 10% per annum compounding semi-annually is equivalent to an annual effective rate of 10.25%. That is, both interest rates are equivalent since they generate the same future value. Hence, a rational individual would be
indifferent to an interest rate of 10% per annum compounding semi-annually or an annual effective interest rate of 10.25%.

In general, suppose you are quoted an annual interest rate of \( r_m \%) compounded \( m \) times per year. For example, \( m=12 \) denotes monthly compounding, \( m=4 \) denotes quarterly compounding, etc. The effective annual interest rate, compounding only once per year, is \( a \) in the following expression:

\[
1 + a = \left(1 + \frac{r_m}{m}\right)^m.
\]

**Example:**

An interest rate of 9\% per annum compounding quarterly is equivalent to an annual effective interest rate of

\[
\left(1 + \frac{0.09}{4}\right)^4 - 1 = 0.09308331879 = 9.308331879\%.
\]

Notice that the annual effective interest rate is numerically larger than the 9\% per annum compounding quarterly interest rate. This is because of the interest on interest effect as those little payments of within-the-year interest themselves earn interest.

The per annum semi-annual, quarterly, monthly and daily compounding interest rates that are all equivalent to the same annual effective rate, \( a \), are given by the following set of equations:

\[
1 + a = \left(1 + \frac{r_2}{2}\right)^2 \quad (m = 2, \text{ that is, semi-annually})
\]

\[
= \left(1 + \frac{r_4}{4}\right)^4 \quad (m = 4, \text{ that is, quarterly})
\]

\[
= \left(1 + \frac{r_{12}}{12}\right)^{12} \quad (m = 12, \text{ that is, monthly})
\]

\[
= \left(1 + \frac{r_{365}}{365}\right)^{365} \quad (m = 365, \text{ that is, daily})^7.
\]

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7 A 365-day year has been assumed here. Some markets operate under other conventions.
These equations clearly show the interplay between the size of the within-the-year interest payments (larger, smaller) and how often they are being paid (less frequently, more frequently) to yield the same annual effective rate, $a$. You should see a pattern in these relationships, especially once you recognise that:

$$1 + a = \left(1 + \frac{r}{1}ight)^m = \left(1 + \frac{r}{1}ight)$$  \hspace{1cm} (m = 1, that is, annual effective).

The Hewlett Packard 17bII+ financial calculator has a set of buttons which convert interest rates between different payment frequencies and also convert to/from continuously compounding rates (see below).

Questions

13. You are quoted an interest rate of 12% per annum compounding quarterly. What is the equivalent annual effective interest rate?
14. You are quoted an interest rate of 12% per annum compounding semi-annually. What is the equivalent per annum interest rate compounding monthly?

In practical terms the compounding frequency, $m$, tells you two key things about how to use that interest rate, $r_m$, in present value and future value calculations:

(1) always use the rate in its per period effective form. Where the per period effective interest rate $r$ is \( \frac{r_m}{m} \); and

(2) measure time not in years (unless $m=1$) but in the periods specified by $m$. 
Question

15. What is the present value of a $1,000 to be received in 5 years if the relevant interest rate is 12% per annum compounding monthly?

Previously we went as far as daily compounding. Suppose we keep pushing this notion and consider more and more frequent compounding. We could go to hourly, per minute, per second, etc. In the limit, we are thinking about really small (infinitesimally small) amounts of interest being paid really, really often (in continuous time). The interest rate which pays in this way is called a continuously compounding interest rate.

If compounding is continuous, then we can use the exponential function, $e^x$, to compute future values. With continuous compounding, the future value of investing $1 for one year is $e^{r_c}$, where $e$ is a constant approximately equal to 2.71828, and $r_c$ is the continuously compounding interest rate.
If we invest $1 for one year at the continuously compounding interest rate that is equivalent to the annual effective rate, $a$, then the future value of that $1 is given by:

$$e^{rc} = 1 + a.$$ 

If we know the future value of the $1 we invested, then we can compute the continuously compounding interest rate by using the natural logarithm, ln. Applying the natural logarithm to both sides of the above equation, we have:8

$$r_c = \ln(1 + a).$$

In general, the per annum continuously compounding interest rate that is equivalent to a quoted per annum interest rate $r_m \%$ compounding $m$ times per year, is $r_c$ in the following:9

$$r_c = \ln(1 + a) = \ln\left(\left(1 + \frac{r_m}{m}\right)^m\right) = m\ln\left(1 + \frac{r_m}{m}\right).$$

---

8 This uses the algebraic fact that: $\ln(e^x) = x$.

9 This uses the algebraic fact that: $\ln(y^x) = x\ln(y)$. 
Questions

16. What is the per annum continuously compounded interest rate that is equivalent to 8% per annum compounding daily? (Assume a 365-day year).

17. What is the daily continuously compounded interest rate that is equivalent to 8% per annum compounding daily? (Assume a 365-day year).
Continuously compounding interest rates may seem a little weird to you. They are(!). But they play an extremely important role in finance, especially in options theory. They are also used extensively by empirical researchers and practising financial analysts because of a property, observed again and again in data, that they are much more symmetrically distributed than are the original once-per-period rates.

Unfortunately we rarely observe continuously compounding rates in practice. They’re certainly not quoted on any screens (Why not?). They are really just another change of payment frequency, \( m \), which we played around with three pages ago. That is, any given annual effective interest rate, \( a \) (for which \( m=1 \)), can be equivalently re-expressed with a different payment frequency (\( m=2, m=12, m=365, \text{ etc.} \)). The equivalent continuously compounding rate has \( m \to \infty \) and is calculated using the \( \ln(\ldots) \) function. Notice that, for a given annual effective rate, the continuously compounding rate, \( r_c \), will be numerically the smallest since it has the greatest interest on interest effect.

**Question**

18. Complete the following table with equivalent rates.

<table>
<thead>
<tr>
<th>Annual rate</th>
<th>( m )</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Annually</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Semi-annually</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quarterly</td>
</tr>
<tr>
<td>12%</td>
<td></td>
<td>Monthly</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Daily</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Continuously</td>
</tr>
</tbody>
</table>

Observe that the table was “seeded” with a single number. All the other rates you calculated are equivalent, in an annual effective sense, to that original rate.
Continuously compounding discount factors and future value factors look different from their per period effective cousins. Suppose \( r_c \) is a per annum continuously compounding interest rate. Then the present value, \( PV \), of a cash flow \( C_T \) to be paid or received \( T \) years from now is:

\[
PV = C_T \times e^{-r_c T}.
\]

Similarly, the future value, \( FV \), in \( T \) years time of a cash flow \( C_0 \) to be paid or received today is:

\[
FV = C_0 \times e^{r_c T}.
\]

Notice that when we calculate present values and future values with continuously compounding interest rates we measure time in years (or fractional years).

**Questions**

19. What is today’s value of $10,000,000 flowing in 3½ years time if the relevant interest rate is 4.75% per annum continuously compounded?

Convert this rate to its equivalent annual effective rate and show that the answer is the same.
20. What is the future value of $5,000,000 (a today amount) in $7\frac{1}{4}$ years time if the relevant interest rate is 2.50% per annum continuously compounded?

Convert this rate to its equivalent annual rate, paying interest quarterly, and show that the answer is the same.
0.6 Problem set

1. Calculate the future value of $10,000 in
   a. 5 years at an annual effective interest rate of 5%.
   b. 10 years at an annual effective interest rate of 5%.
   c. Why is the amount of interest earned in part a) less than half the amount of interest earned in part b)?

2. What is the present value of $10,000 received
   a. 10 years from today when the annual effective interest rate is 5%.
   b. 20 years from today when the annual effective interest rate is 10%.

3. Would you rather receive $5,000 today or $10,000 in 10 years? Assume an annual effective interest rate of 7%.

4. You have won the lottery. Lottery officials offer you the choice of the following alternative payouts:

   Alternative 1: $10,000,000 today.
   Alternative 2: $13,000,000 five years from now.

   What annual effective interest rate will make you indifferent between the two alternatives?

5. You have received a windfall from an investment you made in a friends business. She will be paying you $10,000 at the end of year 1, $20,000 at the end of year 2, and $30,000 at the end of year 3. The relevant annual effective interest rate is 5%.
   a. What is the present value of your windfall?
   b. What is the future value of your windfall in 3 years (on the date of the last payment)?
6. The British government has a consol bond outstanding paying £1,000 per year forever. Assume the current annual effective interest rate is 4%.
   a. What is the value of the consol immediately after a payment is made?
   b. What is the value of the consol immediately before a payment is made?
   c. How much would you pay today to purchase the above consol if the first payment is made in 6-months?

7. A new car valued at $135,000 is to be leased over 5 years. The lessee will purchase the car for $75,000 at the end of the leasing period (i.e., at the end of 5 years). What monthly payments, starting immediately, are necessary to yield the lessor 14% per annum compounding monthly? (Hint: Calculate the payments from the lessor’s point of view).

8. Your company is leasing a machine over 4 years. Monthly payments are $2,400 with the first two payments to be made immediately. The remaining 46 monthly payments start at the beginning of the 2nd month. The residual value of the machine, to be paid at the end of the fourth year, is $15,000. What is the capitalised value (present value) of the lease? Your borrowing rate is 18% per annum compounding monthly. Ignore taxes.

9. You are quoted an annual effective interest rate of 10%. What is the equivalent annual rate compounding semi-annually?

10. Starting today, you make monthly deposits of $250 into an account paying 5% per annum compounding daily (365-day basis). At the end of seven years, how much money will be in the account?