Solutions to Topic 0 embedded questions

1. \[ FV = \$687,436.81(1+0.055)^7 = \$1,000,000 \]
   
   Using your financial calculator:
   Background settings: 1 \( P/YR, END \ MODE \). Then input the following: \( N = 7, I\%YR = 5.5, PV = -687,436.81, PMT = 0 \) and ask for \( FV \).
   A: \$1,000,000

2. There will be a cash outflow of \$687,436.81 today and a cash inflow of \$1,000,000 at the end of year 7.

3. \[ PV = \frac{\$1,000,000}{(1+0.055)^7} = \$687,436.81 \]

   Using your financial calculator:
   Background settings: 1 \( P/YR, END \ MODE \). Then input the following: \( N = 7, I\%YR = 5.5, PMT = 0, FV = 1,000,000 \) and ask for \( PV \).
   A: \$687,436.81

4. \[ r = \left( \frac{\$1,000,000}{\$687,436.81} \right)^{\frac{1}{7}} - 1 = 5.5\% \text{ p.a. annual effective} \]

   Using your financial calculator:
   Background settings: 1 \( P/YR, END \ MODE \). Then input the following: \( N = 7, PMT = 0, PV = -687,436.81, FV = 1,000,000 \) and ask for \( I\%YR \).
   A: 5.5\% \text{ p.a. annual effective}

5. \[ PV = \frac{\$5,000}{0.05} = \$100,000 \]

6. Today:
   Deposit \$100,000 for one year at 5.00\%

   In one year:
   Receive back the deposit of \$100,000 plus \$5,000 interest (5\% \times \$100,000)
   Reinvest the \$100,000 for another year at 5.00\%
   Spend the \$5,000
   You would do this forever
7. \[ PV = \$1,000,000 \times \frac{1}{0.07} \left( 1 - \frac{1}{(1+0.07)^{15}} \right) = \$9,107,914.01 \]

Using your financial calculator:
Background settings: 1 P/YR, END MODE. Then input the following: \( N = 15, I\%YR = 7, PMT = 1,000,000, FV = 0 \) and ask for \( PV \).
A: \$9,107,914.01

8. There will be a cash outflow of \$9,107,914.01 today and cash inflows of \$1,000,000 at the end of years 1 to 15.

9. \[ PV \text{ of perpetuity } 1 = \frac{\$1,000,000}{0.07} = \$14,285,714.29 \]

\[ PV \text{ of perpetuity } 2 = \frac{\$1,000,000}{0.07} \left( \frac{1}{(1+0.07)^{15}} \right) = \$5,177,800.28 \]

\[ PV \text{ of perpetuity } 1 - PV \text{ of perpetuity } 2 = \$14,285,714.29 - \$5,177,800.28 = \$9,107,914.01 \]

10. The present value is \$1,000,000 (the first payment received today) plus the present value of the remaining 14 payments.

\[ PV = \$1,000,000 + \$1,000,000 \times \frac{1}{0.07} \left( 1 - \frac{1}{(1+0.07)^{14}} \right) = \$9,745,467.99 \]

Using your financial calculator:
Background settings: 1 P/YR, BEGIN MODE. Then input the following: \( N = 15, I\%YR = 7, PMT = 1,000,000, FV = 0 \) and ask for \( PV \).
A: \$9,745,467.99

11. \[ PV = \$1,000,000 \times \frac{1}{0.07} \left( (1 + 0.07)^{15} - 1 \right) = \$25,129,022.01 \]

Using your financial calculator:
Background settings: 1 P/YR, END MODE. Then input the following: \( N = 15, I\%YR = 7, PV = 0, PMT = 1,000,000, \) and ask for \( FV \).
A: \$25,129,022.01
12. 

\[ PV \text{ of an } N \text{ period annuity} = \frac{FV \text{ of an } N \text{ period annuity}}{(1 + r)^N} \]

\[ = \frac{25,129,022.01}{(1 + 0.07)^{15}} \]

\[ = 9,107,914.01 \]

\[ FV \text{ of an } N \text{ period annuity} = PV \text{ of an } N \text{ period annuity}(1 + r)^N \]

\[ = 9,107,914.01(1 + 0.07)^{15} \]

\[ = 25,129,022.01 \]

13. Equivalent interest rates generate the same future values and present values. Hence, the future value of investing $1 for one year at 12% per annum compounding quarterly must be the same as the future value of investing $1 for one year at the annual effective rate, \( a \).

Solve for \( a \):

\[ 1 + a = \left(1 + \frac{0.12}{4}\right)^4 \]

\[ a = 12.5509\% \text{ annual effective} \]

14. Solve for \( r_{12} \):

\[ \left(1 + \frac{r_{12}}{12}\right)^{12} = \left(1 + \frac{0.12}{2}\right)^2 \]

\[ r_{12} = 11.7106\% \text{ per annum compounding monthly} \]

15. Here, the number of periods is 60 (5-years \( \times \) 12 compounding periods per year). Therefore, the present value is:

\[ PV = \frac{1,000}{\left(1 + \frac{0.12}{12}\right)^{60}} = 550.45 \]

16. \( r_c = \ln \left(1 + \frac{0.08}{365}\right)^{365} = 365 \ln \left(1 + \frac{0.08}{365}\right) = 7.9991\% \text{ per annum continuously compounded} \)
17. \( r_e = \ln \left(1 + \frac{0.08}{365}\right) = 0.0219\% \) daily (continuously compounded)

18.

<table>
<thead>
<tr>
<th>Annual rate</th>
<th>( m )</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.6825%</td>
<td>1</td>
<td>Annually</td>
</tr>
<tr>
<td>12.3040%</td>
<td>2</td>
<td>Semi-annually</td>
</tr>
<tr>
<td>12.1204%</td>
<td>4</td>
<td>Quarterly</td>
</tr>
<tr>
<td>12%</td>
<td>12</td>
<td>Monthly</td>
</tr>
<tr>
<td>11.9424%</td>
<td>365</td>
<td>Daily</td>
</tr>
<tr>
<td>11.9404%</td>
<td>( \infty )</td>
<td>Continuously</td>
</tr>
</tbody>
</table>

19. Using continuous compounding:
   \( PV = \$10,000,000e^{-0.0475 \cdot 3.5} = \$8,468,344.99 \)

   We have \( \ln(1 + a) = 0.0475 \)

   After applying the exponential function to both sides of the above equation, we have:

   \[
   1 + a = e^{0.0475}.
   \]

   Therefore, \( a = e^{0.0475} - 1 = 4.8646\% \) annual effective

   Using discrete compounding: \( PV = \frac{\$10,000,000}{(1 + 0.048646)^{3.5}} = \$8,468,344.99 \)

20. Using continuous compounding: \( FV = \$5,000,000e^{0.025 \cdot 7.25} = \$5,993,574.10 \)

   We have \( 4 \ln \left(1 + \frac{r_a}{4}\right) = 0.025 \) or \( \ln \left(1 + \frac{r_a}{4}\right) = \frac{0.025}{4} \)

   After applying the exponential function to both sides of the above equation, we have:

   \[
   1 + \frac{r_a}{4} = e^{0.025/4}.
   \]

   Therefore, \( r_a = 4 \left(e^{0.025/4} - 1\right) = 2.5078\% \) per annum compounding quarterly

   Using discrete compounding:

   \[
   FV = \$5,000,000 \left(1 + \frac{0.025078}{4}\right)^{7.25 \cdot 4} = \$5,993,574.10
   \]