Solutions to Topic 0 problems set

1. 
   a) \( FV = 10,000(1+0.05)^5 = 12,762.82 \)

   Using your financial calculator:
   Background settings: 1 \( P/YR \), \( END \) MODE. Then input the following: \( N = 5, I\% YR = 5, PV = -10,000, PMT = 0 \) and ask for \( FV \).
   \( A: \ 12,762.82 \)

   b) \( FV = 10,000(1+0.05)^{10} = 16,288.95 \)

   Using your financial calculator:
   Background settings: 1 \( P/YR \), \( END \) MODE. Then input the following: \( N = 10, I\% YR = 5, PV = -10,000, PMT = 0 \) and ask for \( FV \).
   \( A: \ 16,288.95 \)

   c) Because in the last 5 years you get interest on the interest earned in the first 5 years as well as interest on the original $10,000.

2. 
   a) \( PV = \frac{10,000}{(1+0.05)^{10}} = 6,139.13 \)

   Using your financial calculator:
   Background settings: 1 \( P/YR \), \( END \) MODE. Then input the following: \( N = 10, I\% YR = 5, PMT = 0, FV = 10,000 \) and ask for \( PV \).
   \( A: \ 6,139.13 \)

   b) \( PV = \frac{10,000}{(1+0.10)^{20}} = 1,486.44 \)

   Using your financial calculator:
   Background settings: 1 \( P/YR \), \( END \) MODE. Then input the following: \( N = 20, I\% YR = 10, PMT = 0, FV = 10,000 \) and ask for \( PV \).
   \( A: \ 1,486.44 \)
3. The decision involves comparing the present value of each alternative. Choose the alternative with the highest present value.

The present value of $5,000 to be received today is $5,000.

The present value of $10,000 to be received in 10 years is \( \frac{10,000}{(1 + 0.07)^{10}} = 5,083.49 \).

You would choose to receive $10,000 in 10 years.

4. In order to indifferent between the two alternatives, the future value of $10,000,000 in 5 years must be $13,000,000. Alternatively, the present value of $13,000,000 received in 5 years must be $10,000,000.

That is, \( 10,000,000(1 + r)^5 = 13,000,000 \). Solving for the annual effective interest rate \( r \), we find \( r = \left( \frac{13,000,000}{10,000,000} \right)^{\frac{1}{5}} - 1 = 5.3874\% \).

Using your financial calculator:
Background settings: 1 P/YR, END MODE. Then input the following: \( N = 5, PMT = 0, PV = -10,000,000, FV = 13,000,000 \) and ask for \( I\%YR \).
A: 5.3874\% per annum annual effective

5. 

a) \( PV = \frac{10,000}{(1 + 0.05)^1} + \frac{20,000}{(1 + 0.05)^2} + \frac{30,000}{(1 + 0.05)^3} = 53,579.527... \)

b) \( FV = 10,000(1 + 0.05)^2 + 20,000(1 + 0.05) + 30,000 = 62,025 \)

Or \( FV = 53,579.527...(1 + 0.05)^3 = 62,025 \)
6.

a) The value of the bond is equal to the present value of the cash flows. By the perpetuity formula:
\[ PV = \frac{1,000}{0.04} = £25,000 \]

b) The value of the bond is equal to the present value of the cash flows. The cash flows are the perpetuity plus the payment that will be received immediately.
\[ PV = \frac{1,000}{0.04} + 1,000 = £26,000 \]

c) The value of the perpetuity immediately before the next payment date in 6-months is £26,000. Today, you would pay the present value of £26,000, which is
\[ \frac{26,000}{(1 + .04)^{\frac{6}{12}}} = £25,495.10. \]

7. Solve for the monthly payments \( C \) in

\[ $135,000 = C + C \left( \frac{1}{0.14/12} \left( 1 - \frac{1}{(1 + 0.14/12)^{60}} \right) \right) + \frac{$75,000}{(1 + 0.14/12)^{60}} \]

\[ C = $2,244.90 \]

Using your financial calculator:

Background settings: 1 P/YR, BEGIN MODE. Then input the following: \( N = 60, I\%YR = 14/12 = 1.16666666667, PV = -135,000, FV = 75,000 \) and ask for \( PMT \). Notice that I use for \( I\%YR \) the interest rate per payment period (per month in this case), which is 14/12. If you have your background settings at 12 P/YR then the \( I\%YR \) input becomes 14. All other inputs remain the same. This means the P/YR setting only interacts with how you input your \( I\%YR \) -- it's an attempt by the calculator to be user-friendly. But it can cause confusion, which is why I always leave my P/YR set to 1 and explicitly work out the interest rate per payment period.

A: $2,244.90
8. \[ PV = 2 \times 2,400 + 2,400 \left( 1 - \frac{1}{1 + 0.18/12}^{46} \right) + \frac{15,000}{(1 + 0.18/12)^{48}} \]
   \[ = 91,476.00 \]

Using your financial calculator:

Background settings: 1 P/YR, BEGIN MODE. Then input: \( N = 47, I\%\text{YR} = 18/12 = 1.50, PMT = -2,400, FV = 0 \) and ask for \( PV \). This yields
   \( PV = 81,735.57591 \). This is the value of 47 payments of $2,400 each, at the beginning of each month. Another $2,400 at \( t = 0 \) (this is the 48th payment) increases this to 84,135.57591. Now explicitly add the PV of the $15,000 residual payment at the end of the 48th month. This PV is \( 15,000/(1.015)^{48} = 7,340.425430 \). Adding this to the previous yields the answer.
   A: $91,476.00

9. Solve for \( r_2 \):

\[ 1 + 0.10 = \left(1 + \frac{r_2}{2}\right)^2 \]

\( r_2 = 9.76177\% \) per annum compounding semi-annually

10. First convert that 5.00\% per annum daily compounded interest rate into its monthly equivalent rate. That is, solve for \( r_{12} \):

\[ \left(1 + \frac{r_{12}}{12}\right)^{12} = 1 + \frac{0.05}{365} \]

That answer is 5.01008724\% per annum compounding monthly.

Background settings: 1 P/YR, BEGIN MODE. Input \( N = 84, I\%\text{YR} = 5.01008724/12 = 0.41750727, PV = 0, PMT = 250 \) and ask for \( FV \).
   A: $25,196.15